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Soft SUSY breaking and family symmetry

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Abstract: A spontaneously broken non-abelian SU(3) family symmetry can generate a realistic form for quark, charged lepton and neutrino masses and mixing angles. It also gives a new solution to the SUSY flavour problem by ensuring near family degeneracy of the soft mass SUSY breaking terms. However the need to generate large third generation fermion masses means that the group must be strongly broken to SU(2) giving significant corrections to the third family squark and slepton masses. We investigate the phenomenological implications of such breaking and show that it leads to new solutions capable of fitting all present experimental measurements and bounds as well as the dark matter abundance.

Keywords: Supersymmetry Breaking, Supersymmetry Phenomenology, Cosmology of Theories beyond the SM, Supersymmetric Standard Model.

Contents

1. Introduction

The origin of fermion masses and mixings is perhaps the most pressing of the questions left unanswered by the Standard Model. Particularly noticeable is the difference between the mixing angles in the quark and lepton sectors. In the quark sector the mixing angles are small. However recent measurements of neutrino oscillation have shown that the mixing of the atmospheric neutrinos is consistent with being bi-maximal with equal components in the ν_μ, ν_τ directions while that of the solar neutrinos is consistent with being tri-maximal with equal components in the ν_e, ν_μ, and ν_τ directions.

In [1] it was argued that this data suggests the existence of an underlying non-abelian family symmetry capable of relating the coupling of the Higgs to different families. To demonstrate this an SU(3) family symmetry, the largest consistent with an underlying SO(10) Grand Unified symmetry, was constructed and shown to be capable of describing both the quark and lepton masses and mixings. In order to generate the third generation quark and charged lepton masses the family symmetry must be strongly broken to SU(2). The first two generation quark and charged lepton masses are generated by a second stage of breaking which preserves a discrete subgroup of the SU(3) having matrix elements which are equal in the 2 and 3 directions. With this breaking scheme, the effective Yukawa couplings constrained by additional abelian symmetries leads to mixing angles in the up and down quark (and charged lepton) sectors which are small and in agreement with the measured values. Although the Dirac mass of the neutrinos has the same general form as that of the quarks and leptons the light neutrinos have large mixing angles. This follows from the see-saw mechanism with sequential domination of the right-handed neutrinos. In this the near bi-maximal mixing comes from the correlation in the 2 and 3 directions of the second stage of breaking, a direct consequence of the underlying SU(3) symmetry. The near tri-maximal mixing of the solar neutrino also follows from this vacuum alignment.

JHEP08(2005)031

As stressed in [1] an important byproduct of such a family symmetry is that, while unbroken, it guarantees the degeneracy of a family of squarks or sleptons in a given representation of the Standard Model. As a result it provides a new solution to the "family" problem, the need to have the squarks and charged sleptons nearly degenerate to suppress flavour changing neutral currents and to suppress CP violating effects in dipole electric moments. This solution eliminates the need to appeal to the alternative solutions which communicate supersymmetry breaking to the visible sector via various "mediator" mechanisms, including gravity mediation, gauge mediation and anomaly mediation.

In this paper we will study the phenomenological implications of the SU(3) family symmetry solution to the family problem. We concentrate on the minimal case where an underlying Grand Unified symmetry guarantees the degeneracy of the squarks and sleptons at the unification scale. However the need to generate the third generation of quark and lepton masses requires that there is strong breaking of this degeneracy for the third generation of squarks and sleptons. Although we motivate this study in the context of a specific implementation of a family symmetry model, it seems likely that the structure applies more generally. Any theory of fermion masses must distinguish the third family from the light families and this is likely to have an effect on the third sfamily masses too. We will show that the splitting of the third family of sfermions following from the dominant breaking of the family symmetry leads to significant change in the resulting phenomenology. In particular we find a new class of solutions which satisfy all the current bounds on supersymmetric states, have gauge coupling unification and allow for radiative electroweak breaking, are consistent with present measurements of $b \longrightarrow s\gamma$ and the anomalous magnetic moment of the muon and have an LSP abundance which generates the observed dark matter abundance.

The paper is organised as follows. In section 2 we briefly review the expectation for sfermion and Higgs masses in a supersymmetric theory with an SU(3) family symmetry. We then elaborate on the problem of possible D-term contributions to the scalar masses, which might violate the bounds on FCNC, in section 3. In section 4 we discuss the various components of the global fit and the method used to perform the renormalisation group flow and the analysis of the dark matter abundance. The results are presented in section 5. Finally in section 6 we discuss the implications of the results and summarize.

2. The sparticle spectrum

In a theory with an underlying SO(10) symmetry the soft supersymmetry breaking masses of the states in a single family will be degenerate in the absence of SO(10) breaking. When the symmetry is extended to include a non-abelian SU(3) family symmetry this degeneracy applies to soft supersymmetry breaking masses of all the squarks and sleptons. If, in addition, one assumes the Higgs scalars have the same initial mass one obtains the Constrained Minimal Supersymmetric Standard Model (CMSSM) spectrum often assumed in supergravity models, see for example, [2 − 9].

Of course, in the case of the non-abelian family symmetry solution to the family problem, there is no symmetry reason for assuming the degeneracy of the Higgs scalars.

JHEP08(2005)031

The effect of breaking this degeneracy has been extensively explored in [10 − 13]. Here we wish to explore further differences between the CMSSM and the non-abelian symmetry solution to the family problem. These arise when the GUT and family symmetries are broken and are dependent on the pattern of symmetry breaking.

In [1] the full SO(10) symmetric model was not constructed but it was assumed the SO(10) was broken close to the unification scale to $SU(4) \otimes SU(2)_L \otimes SU(2)_R$. If this is the dominant breaking effect the result is that the Pati Salam group SU(4) preserves the degeneracy of the up squarks and sneutrinos and of the down squarks and sleptons in a given $SU(2)_L \otimes SU(2)_R$ representation. Combined with the SU(3) family symmetry this means that there is degeneracy of the $SU(2)_L(SU(2)_R)$ doublets of squarks and sleptons families as in the CMSSM but there may be breaking between the left- and right- handed states.

Another possibility is that the dominant breaking is that of the SU(3) family symmetry when giving the third family of fermions their masses. In this case an underlying SO(10) symmetry would guarantee the degeneracy of all the states in a given family, but the breaking of SU(3) to SU(2) means that the degeneracy between the first two families and the third family is lost. If SO(10) is broken instead to SU(5) the degeneracy maintained is between the right-handed down squarks and the slepton doublets and, separately, between the right-handed up squarks, the squark doublets and the right-handed charged slepton. However, unlike the CMSSM, these two groups of states can have different masses.

Of course both effects are likely to be present in a realistic theory but the analysis of the general case is very difficult due to the large number of parameters introduced. In this paper we shall explore the implications of the second possibility where the dominant breaking is of the family symmetry but the members of an SO(10) multiplet remain degenerate at the unification scale. We shall explore the more general possibilities elsewhere but we choose to start with this simple case as the breaking of the family symmetry breaks the solution to the family problem and, as discussed above, can lead to significant changes in flavour changing and CP violating processes.

In the model discussed in [1] the dominant soft supersymmetry breaking mass terms come from the $D-$term $m_0^2\,(\psi_i)^\dagger\,\psi_i|_D$ where ψ_i are the quark and lepton supermultiplets, triplets under the SU(3) family symmetry, and m_0^2 is the supersymmetry breaking mass scale in the visible sector. This clearly leads to degeneracy between the three families (we do not need to specify the mediator sector as the degeneracy follows from the family symmetry). The origin of the splitting between family multiplets comes from the D-terms $m_0^2\left(\psi_i\phi^i\right)^\dagger\psi_j\phi^j/M^2|_D$ where the fields ϕ^i are the (SU(3) antitriplet) fields which break the family symmetry. M is the mass of the messenger communicating family symmetry breaking to the quarks and leptons. The dominant breaking comes from the field ϕ_3 which has a vacuum expectation value (vev) $\langle\phi_3^3\rangle = a$. This gives a relative mass difference of $\mathcal{O}(a/M)$ to the third generation. In [1] this was chosen close to unity $a/M \simeq 0.4$, its precise value (< 1) being undetermined. Breaking of $\mathcal{O}(0.1)$ at the unification scale between the third and first two generations must be compared with the bounds on the splitting between families coming from flavour changing neutral currents (FCNC). Such $\mathcal{O}(1)$ breaking of the third family gives a reduced effect on flavour changing bounds due to two effects. Firstly radiative corrections from gauge interactions increase the average quark and slepton masses at low scales without amplifying the breaking between families, thus reducing the relative breaking effects. Secondly the mixing of the third family to the light generations is small. As a result the dominant breaking of the family symmetry is consistent with the precision bounds coming from flavour changing processes [14].

The second breaking of the family symmetry is due to the field ϕ_{23} which has a vev $\langle\phi_{23}^3\rangle = \langle\phi_{23}^2\rangle = b$. The corresponding soft mass breaking term gives a relative mass difference of $\mathcal{O}(b/M)$ between the first two generations. The fit to the light quark spectrum requires $b/M = \mathcal{O}(0.02)$ in the down sector and $\mathcal{O}(0.003)$ in the up sector. Such splitting is within the bounds of $\mathcal{O}(1\%)$ on the breaking of the first two generations coming from FCNC.

Finally what about the remaining soft supersymmetry breaking parameters? The A terms arise from $m_0 W|_A$ (W is the superpotential) which are generated on supersymmetry breaking in the visible sector through a $m_0\theta\theta$ spurion (θ is the superspace coordinate). They are the same in the effective theory coming from an underlying family symmetry as in the CMSSM. This is because they arise from the effective Yukawa couplings which are responsible for fermion masses and the choice of family symmetry breaking has been dictated by the need to get viable fermion masses as is assumed in the CMSSM.

In this section we have assumed that the only source of non-universality in the sfermion sector arises from the non-renormalisable term, $m_0^2\left(\psi_i\phi^i\right)^\dagger\psi_j\phi^j/M^2|_D$ when the SU(3) family symmetry is broken. However, this is not necessarily the case. We will now address the issue of possible additional contributions to the sfermion soft masses that potentially violate the family symmetry solution to the flavour problem.

3. The effect of D-terms

There is a long standing problem associated with family symmetries in supersymmetric theories coming from the family symmetry D-terms. As originally noted by Murayama et. al. [15, 16], since the quarks and leptons of different generations necessarily have different family charges, the soft supersymmetry breaking masses generated by the D-terms for squarks and sleptons are not degenerate and can lead to unacceptably large flavour changing effects.

The simplest example of this effect is when a U(1) gauge symmetry is broken along a flat direction by charge $+1$ and -1 fields ϕ_+ and ϕ_-. The expectation values of ϕ_+ and ϕ_- can differ if their SUSY masses are different, $m_+^2 \neq m_-^2$:

$$\langle D\rangle = \langle|\phi_+|^2 - |\phi_-|^2\rangle = \frac{1}{g^2}(m_+^2 - m_-^2) \tag{3.1}$$

where g is the gauge coupling constant. Then the D-term gives contributions to the masses of the light scalar fields ρ_i,

$$V = \sum_i g^2\langle D\rangle Q_i|\rho_i|^2 \tag{3.2}$$

JHEP08(2005)031

where Q_i are the charges of the ρ_i fields. Note that this contribution does not decouple even if the U(1) breaking scale is very high. Moreover it is independent of the gauge coupling and cannot be suppressed by taking the coupling small. There is a completely analogous effect for the non-abelian case.

A quantitative measure of the problem is given by the phenomenological requirement on squark and slepton masses coming from the smallness of bounds on quark and lepton flavour changing processes which give [17] for the masses evaluated at a high (unification or string) scale

$$\left|\frac{\tilde{m}_s^2 - \tilde{m}_d^2}{\tilde{m}_s^2 + \tilde{m}_d^2}\right|, \qquad \left|\frac{\tilde{m}_\mu^2 - \tilde{m}_e^2}{\tilde{m}_\mu^2 + \tilde{m}_e^2}\right| \lesssim 10^{-}1\,. \tag{3.3}$$

Note that there are much stronger limits for these mass differences at low scales but these are weakened at high scales due to the universal contribution to the soft masses at low scales coming from gaugino masses. From this one may see that the D-terms will not give unacceptable contributions to flavour changing processes if $|m_+^2 - m_-^2| < 0.1(|\tilde{m}_s^2 + \tilde{m}_d^2|, |\tilde{m}_\mu^2 + \tilde{m}_e^2|)$. There are several ways this may happen:

1. If the initial masses are degenerate, as happens in various mechanisms of supersymmetry breaking, the $|m_+^2 - m_-^2|$ will arise only through radiative effects if ϕ_+ and ϕ_- have different Yukawa couplings. In specific models the symmetries of the model can prevent this, readily leading to the bound of Eq. 3.3 being satisfied. Even if there are different couplings for ϕ_+ and ϕ_-, if the scale of family symmetry breaking is close to cut-off scale (the string or the Planck scale), the radiative effects may readily be small as they will not be enhanced by large logs because the running of the masses giving rise to these logarithmic enhancements is cut off at the family symmetry breaking scale. In this case again one readily satisfies the bounds of Eq. 3.3.

2. If the initial masses are not degenerate there is clearly a potential problem because ϕ_+ and ϕ_- need not be degenerate. However in this case the question whether D-terms cause problems is dependent on the fine details of the SUSY breaking mechanism for it may readily happen that m_+ and m_- are less than $\frac{1}{3}$ of $\tilde{m}_s$, $\tilde{m}_d$, $\tilde{m}_\mu$ and $\tilde{m}_e$ and consistent with the bounds of Eq. 3.3. A specific example of this occurs when the SUSY breaking mediator mass M is less than the family symmetry breaking mass scale $< \phi_+ >$. In this case there is a suppression of m_+ and m_- because the radiative graphs generating these masses via the mediator will be suppressed by a factor of order $\frac{M}{<\phi_+>}$ due to the fact that these graphs will always involve states of mass of $\mathcal{O}(< \phi_+ >)$. There will be no such suppression for the squark and slepton masses because the associated radiative effects only involve states lighter than the mediator mass

3. Finally it may be that the family symmetry is only a discrete gauge subgroup of the underlying continuous family symmetry group (abelian or non-abelian). In this case there are no D-terms so the problem disappears [18]. In our analysis we have implicitly assumed the family symmetry group, SU(3), is continuous but our results apply to the case that the family group is a discrete subgroup of SU(3) too. The

reason is because, apart from the D-terms, the difference between the continuous and discrete cases is due to the appearance of new terms in the Lagrangian allowed by the discrete symmetry. Typically (depending on the size of the discrete subgroup) these terms will appear at higher non-renormalisable order and so will not affect the structure of the soft terms assumed here.

We will assume that at least one of these solutions holds, which is indeed the case for the model investigated in [1] upon which this analysis is based. We shall now proceed to investigate the phenomenological implications of the breaking of the family degeneracy in the soft SUSY breaking masses coming from the pattern of symmetry breaking needed to generate the fermion masses. In practice the breaking between the first two families is a negligible change to the CMSSM boundary conditions but the splitting of the third family can be significant. Although we have motivated this study in the context of a specific model, the range of splitting we consider is dictated by the observed fermion mass spectrum and so is likely to be the same for any theory with an underlying family symmetry ordering the fermion masses which is consistent with the FCNC bounds.

4. Calculation and constraints

In what follows we investigate the phenomenological implications of the modification of the CMSSM spectrum discussed above which corresponds to the case of a broken family symmetry. In particular we compare the case of a degenerate scalar spectrum at the unification scale to one in which the mass squared of the third generation of squarks and charged sleptons is allowed to vary by up to 20%.

We use `SOFTSUSY` v.1.8.7 [19], one of several publicly available codes, to calculate the sparticle spectrum and mixings. The code has been augmented to include our family symmetry-inspired boundary conditions and a routine for the calculation of the SUSY contribution to the muon anomalous magnetic moment using the formulae in [20]. `SOFTSUSY` uses a bottom-up routine in which various low energy observables such as M_Z, fermion masses and gauge couplings are input as constraints in addition to the GUT scale boundary conditions. An iterative algorithm proceeds from an initial guess to find a set of sparticle masses and mixings consistent with the high and low scale constraints. We use full 2-loop renormalization group equations for the gauge and Yukawa couplings and the μ parameter. For the soft masses we use the full 1-loop RGEs and include the 2-loop contributions in the 3rd family approximation. Full details can be found in [19]. A comparison between `SOFTSUSY` and similar programs, for example [21–23], was made in [24] and one can directly compare the codes online at [25].

For the calculation of the neutralino relic density and $\mathcal{B}(b \to X_s\gamma)$ we use `micrOMEGAs` v.1.3.1 [26], linked to `SOFTSUSY` via an interface conforming with the Les Houches Accord [27] standard that contains all the relevant parameters from `SOFTSUSY` necessary for the relic density calculation. For details of these calculations, see [26] and the papers on which they were based [9, 28–36].

In our analysis we impose the following constraints:

JHEP08(2005)031

Direct searches The following lower limits from LEP provide the strongest constraints on sparticle masses from direct searches [37]:

$$m_{\tilde{\chi}^{\pm}} \geq 103\,\mathrm{GeV} \qquad m_{\tilde{e}_R} \geq 99\,\mathrm{GeV}\,.$$

We include these lower bounds in our plots.

Muon anomalous magnetic moment We include the 2σ bounds on the discrepancy between experiment and Standard Model theory assuming the latest results of the calculation based on e^+e^- data for the hadronic contribution [38] and the most recent data from the BNL E821 experiment incorporating the results from negative muons [39]. We use the values from [40] which include the recently recalculated α^4 QED correction [41] and the most recent hadronic light-by-light contribution [42]. Similar values were obtained by an independent calculation [43]. However this second paper does not take the new theoretical results [41, 42] into account. From [40],

$$a_\mu^{exp} - a_\mu^{SM} = (24.5 \pm 9.0) \times 10^{-10}\,,$$

where $a_\mu \equiv \frac{(g-2)_\mu}{2}$. We use the 2σ bound,

$$6.5 \times 10^{-10} < \delta a_\mu < 42.5 \times 10^{-10}\,,$$

as the allowed range of the SUSY contribution. Due to the inconsistency between these results and those obtained by using τ decay data, and taking into account the susceptibility to change of the measurement of the e^+e^- cross section [38], the $(g-2)_\mu$ constraint should perhaps be viewed more provisionally than the others. This is unfortunate since it is one of the most important, being the only one that unambiguously determines the sign of μ.

Branching Ratio $\mathcal{B}(\mathbf{b} \to \mathbf{X_s}\gamma)$ The most recent world average for the branching ratio is [44]

$$\mathcal{B}(b \to X_s\gamma)_{exp} = (3.34 \pm 0.38) \times 10^{-4}\,,$$

while the current Standard Model theory value is [45][1]

$$\mathcal{B}(b \to X_s\gamma)_{SM} = (3.70 \pm 0.30) \times 10^{-4}\,.$$

We will use this Standard Model estimate of the theoretical error in our calculation as representative of the error to be expected in our calculation which includes both Standard Model and SUSY contributions. We do this by combining the experimental and theoretical errors in quadrature to obtain the following upper and lower bounds on the branching ratio at 2σ:

$$2.40 \times 10^{-4} < \mathcal{B}(b \to X_s\gamma) < 4.28 \times 10^{-4}\,.$$

[1] This takes into account only those results that include the improved ratio $m_c^{\overline{MS}}(m_b/2)/m_b^{pole}$ as opposed to m_c^{pole}/m_b^{pole} in the $\langle X_s\gamma|(\bar{s}c)_{V-A}(\bar{c}b)_{V-A}|b\rangle$ matrix element. For details see [31].

JHEP08(2005)031

Neutralino dark matter The analysis of the data from WMAP gives a best fit value for the matter density of the universe of $\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$ and for the baryon density, $\Omega_b h^2 = 0.0224 \pm 0.0009$ [46]. This implies that the CDM density is

$$\Omega_{CDM} h^2 = 0.1126^{+0.0161}_{-0.0181}$$

at the 2σ level. This can be an extremely stringent bound on the MSSM parameter space, especially in the case of small $\tan\beta$. However, for large $\tan\beta$ it is less restrictive due to the presence of the A^0 Higgs resonance, and much less so if we allow for a source of cold dark matter other than neutralinos such as axions, or some relic density enhancement mechanism such as non-thermal production of neutralinos (see [47] and references therein for more examples). In these instances, the lower bound on $\Omega_m h^2$ can be neglected. We plot values for which

$$0.0945 < \Omega_{CDM} h^2 < 0.1287\,,$$

and indicate the allowed regions if we choose to discard the lower bound. We also plot the locus of points for which $m_{A^0} = 2m_{\tilde{\chi}^0_1}$ marking the position of the A^0 resonance.

Lightest Higgs Mass $\mathbf{m_{h^0}}$ We also display the contour

$$m_{h^0} = 114.1\,\mathrm{GeV}\,,$$

corresponding to the LEP bound on the lightest SM Higgs boson [37] in the regions of parameter space where the lightest MSSM Higgs boson is Standard Model like, i.e. $\sin(\beta-\alpha)$ is almost exactly equal to 1, where α is the mixing angle relating the mass eigenstates to the gauge eigenstates in the CP-even neutral Higgs sector. This condition applies throughout the parameter space we analyse here.

Correct EWSB / Tachyons / Higgs potential unbound from below The boundary on which $|\mu|^2$ vanishes, marking the border of correct radiative electroweak symmetry breaking has been plotted. In the region where $|\mu|^2 < 0$ a global minimum of the two loop effective Higgs potential cannot be found. Similarly, any regions in which $m^2_{A^0} < 0$, also signalling that the electroweak symmetry has not been broken correctly, have been excluded. Regions with tachyonic sfermions are likewise omitted.

5. Results and discussion

We first present our results for universal boundary conditions, i.e. all scalar masses are set to m_0 and all gaugino masses to $m_{1/2}$ at the GUT scale. Figure 1 shows the $(m_{1/2}, m_0)$ plane of the CMSSM for $\tan\beta = 10$, 30 and 50 for plots a, b and c respectively. We set $\mu > 0$ in accordance with the expectation for $(g-2)_\mu$, and $A_0 = 0$ for simplicity. Our plots can be seen to be in reasonable agreement with those in recent papers [48–54].[2]

[2] Any discrepancies between the results shown in figure 1 and those of other papers are likely to be due to the differing approximations used to compute the sparticle spectrum and mixings and the neutralino relic density. One can find comparisons between various commonly used codes in ref. [24].

JHEP08(2005)031

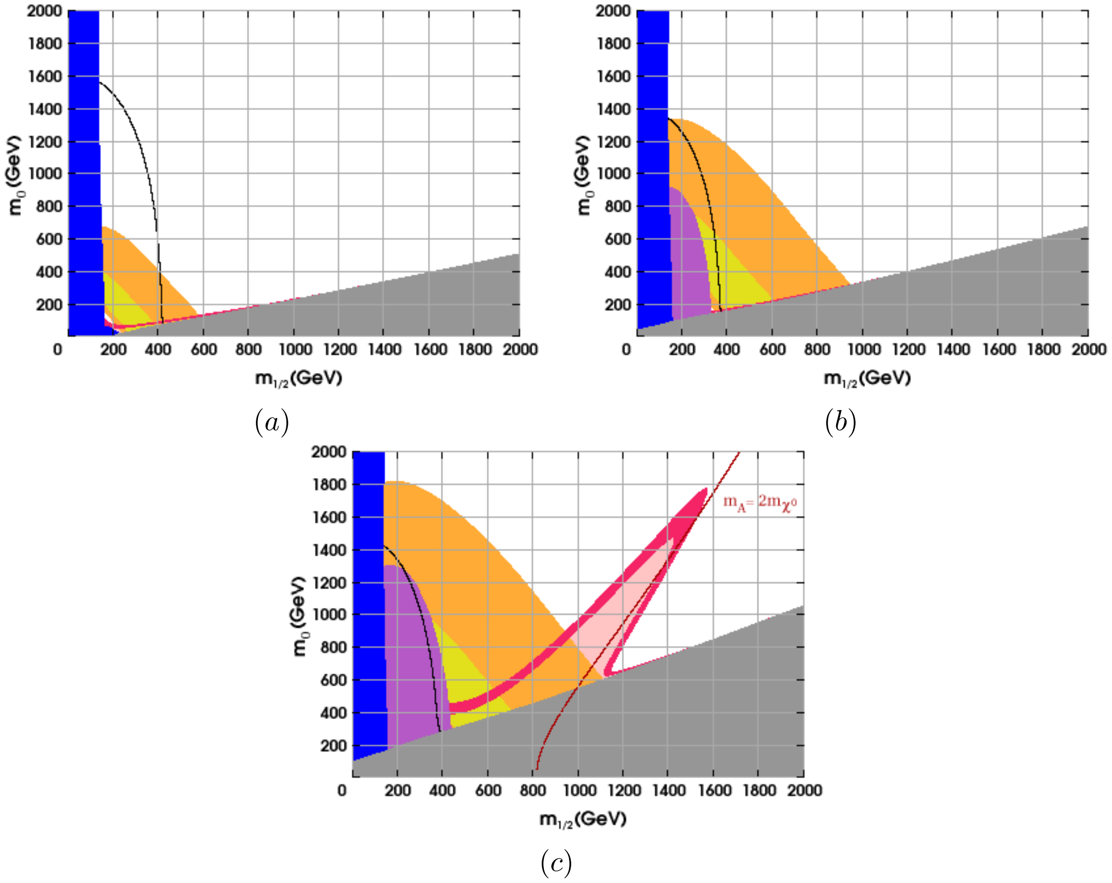


Figure 1: The $(m_{1/2}, m_0)$ plane with $\mu > 0$, $A_0 = 0$ in the CMSSM for (a) $\tan\beta = 10$, (b) $\tan\beta = 30$ and (c) $\tan\beta = 50$. The medium grey region is excluded because the LSP is a stau, the blue(v. dark grey) region is excluded by the LEP limits on sparticle masses and $\mathcal{B}(b \to X_s\gamma)$ is too small in the lilac(darkish grey) region; the black line corresponds to the contour $m_{h^0} = 114.1$ GeV; the red(dark grey) strip shows where $0.0945 < \Omega_{CDM}h^2 < 0.1287$ and the yellow(v. light grey) and orange(light grey) regions represent the 1σ and 2σ bounds on the region favoured by $(g-2)_\mu$. The position of the A^0 resonance where $2m_{\tilde{\chi}_1^0} = m_{A^0}$ is shown as a dark red (dark grey) line and regions satisfying only the upper bound on $\Omega_{CDM}h^2$ are shaded light pink (v. light grey).

We focus our discussion on the neutralino relic density. There is a narrow band for each value of $\tan\beta$ where the bounds on $\Omega_{CDM}h^2$ and $(g-2)_\mu$ are satisfied. In this region, for $\tan\beta = 10, 30$, the main annihilation channels for the neutralinos are t-channel sfermion exchanges to leptons and quarks with coannihilations with staus becoming important close to where $m_{\tilde{\chi}_1^0} = m_{\tilde{\tau}}$. For $\tan\beta = 50$ the main channels in the favoured region are again t-channel sfermion exchanges, but also s-channel A^0 Higgs exchanges. Rapid annihilation via the A^0 dominates as the resonance is approached. Near to the band of parameter space excluded by the chargino mass, for $\tan\beta = 30, 50$, there is a narrow filament of acceptable relic density in the favoured region corresponding to the h^0 resonance. The focus point region [55–58] where $|\mu|$ becomes very small, resulting in a large Higgsino component of the LSP, occurs at a higher value of m_0 than shown in these plots. The boundary where $|\mu| = 0$ begins at $m_0 \simeq 2300$ GeV for $m_{1/2} = 100$ GeV and $\tan\beta = 50$. In this sector of

JHEP08(2005)031

parameter space annihilation to gauge bosons is enhanced and chargino coannihilation also becomes important resulting in an acceptable relic density. However, this region is well outside the range favoured by $(g-2)_\mu$.

We now compare these results with those predicted by the SU(3) family symmetry. Since we do not know the sign of the correction to the third family sfermion masses we consider two additional cases. Figure 2 is the same plot as figure 1, but with the soft supersymmetry breaking sfermion masses squared taking the following form at the GUT scale:

$$m^2_{\tilde{Q}}(m_G) = m^2_{\tilde{u}_R}(m_G) = m^2_{\tilde{d}_R}(m_G) = m^2_{\tilde{L}}(m_G) = m^2_{\tilde{e}_R}(m_G) = m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\delta m^2 \end{pmatrix},$$

and the same for figure 3, but with

$$m^2_{\tilde{Q}}(m_G) = m^2_{\tilde{u}_R}(m_G) = m^2_{\tilde{d}_R}(m_G) = m^2_{\tilde{L}}(m_G) = m^2_{\tilde{e}_R}(m_G) = m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+\delta m^2 \end{pmatrix}$$

where $\delta m^2 = 0.2$ is the correction coming from the SU(3) family symmetry. Although the model also predicts small corrections to the (1,1) and (2,2) elements of the sfermion mass matrices, they have a negligible effect on the phenomenology so we will ignore them.

From figures 2 and 3, it is clear that altering the universal boundary conditions in this way has a relatively small effect on the regions favoured by $(g-2)_\mu$ or on those excluded by $\mathcal{B}(b \to X_s\gamma)$ or direct search constraints and we will not discuss these effects further. Due to the variation of the slepton soft mass matrices there is a small change in the slope of the boundary $m_{\tilde{\chi}_1^0} = m_{\tilde{\tau}}$ in the three different cases. This has a rather small effect on the preferred region of parameter space. However, there is a very significant change in the region of allowed electroweak symmetry breaking and the neutralino relic density in the case of decreased third family soft sfermion masses as shown in figure 2, especially for large $\tan\beta$. As we shall see, this is predominantly due to the change in the third family squark soft masses, and is relatively insensitive to changes in the slepton sector.

One of the main features of the plots in figure 2 is the large region where the electroweak symmetry is not broken correctly. Here, $|\mu|^2 < 0$ is found when the minimisation conditions are applied to the scalar potential indicating that an acceptable minimum cannot be found. The boundary of this region corresponds to $|\mu| = 0$. The effect of decreasing the third family sfermion soft masses on EWSB can be understood by considering the expression for $|\mu|^2$ from the EWSB conditions and the RGE for $m^2_{H_2}$. Including quantum corrections [59],

$$\begin{aligned} |\mu|^2 &= \frac{\overline{m}^2_{H_1} - \overline{m}^2_{H_2}\tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}m_Z^2 - \frac{1}{2}\,\mathrm{Re}\,\Pi^T_{ZZ} \\ &\simeq \frac{\overline{m}^2_{H_1}}{\tan^2\beta} - \overline{m}^2_{H_2} - \frac{1}{2}m_Z^2 - \frac{1}{2}\,\mathrm{Re}\,\Pi^T_{ZZ} \end{aligned}$$

since $\tan\beta \gg 1$. Here, $\overline{m}^2_{H_i} = m^2_{H_i} - t_i/v_i$ where t_i/v_i are the tadpole contributions, and Π^T_{ZZ} is the transverse part of the Z self-energy. This implies the following condition

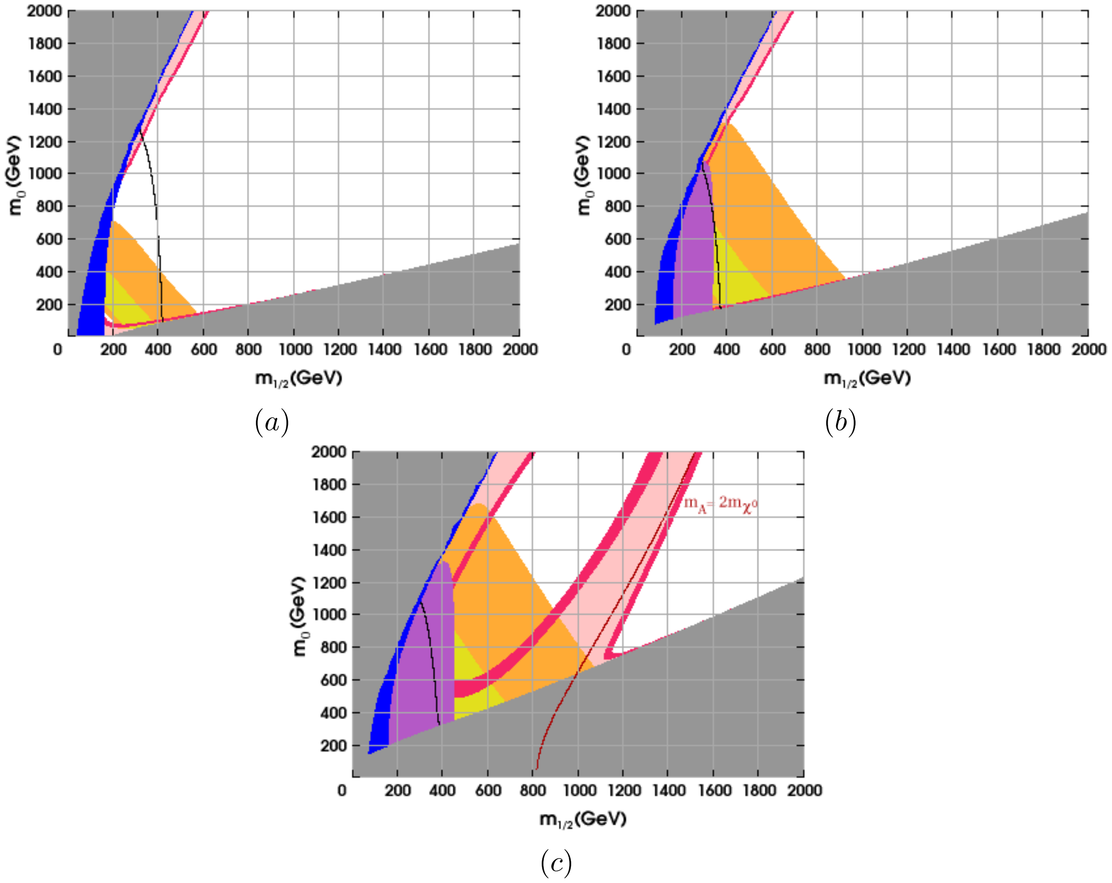


Figure 2: Same as figure 1, but with $m^2_{\tilde{Q}}(m_G) = m^2_{\tilde{u}_R}(m_G) = m^2_{\tilde{d}_R}(m_G) = m^2_{\tilde{L}}(m_G) = m^2_{\tilde{e}_R}(m_G) = m_0^2 \mathrm{diag}(1, 1, 1 - \delta m^2)$. The new medium grey region on the left hand side is excluded by EWSB requirements, i.e. $|\mu|^2 < 0$.

on $\overline{m}^2_{H_2}$:

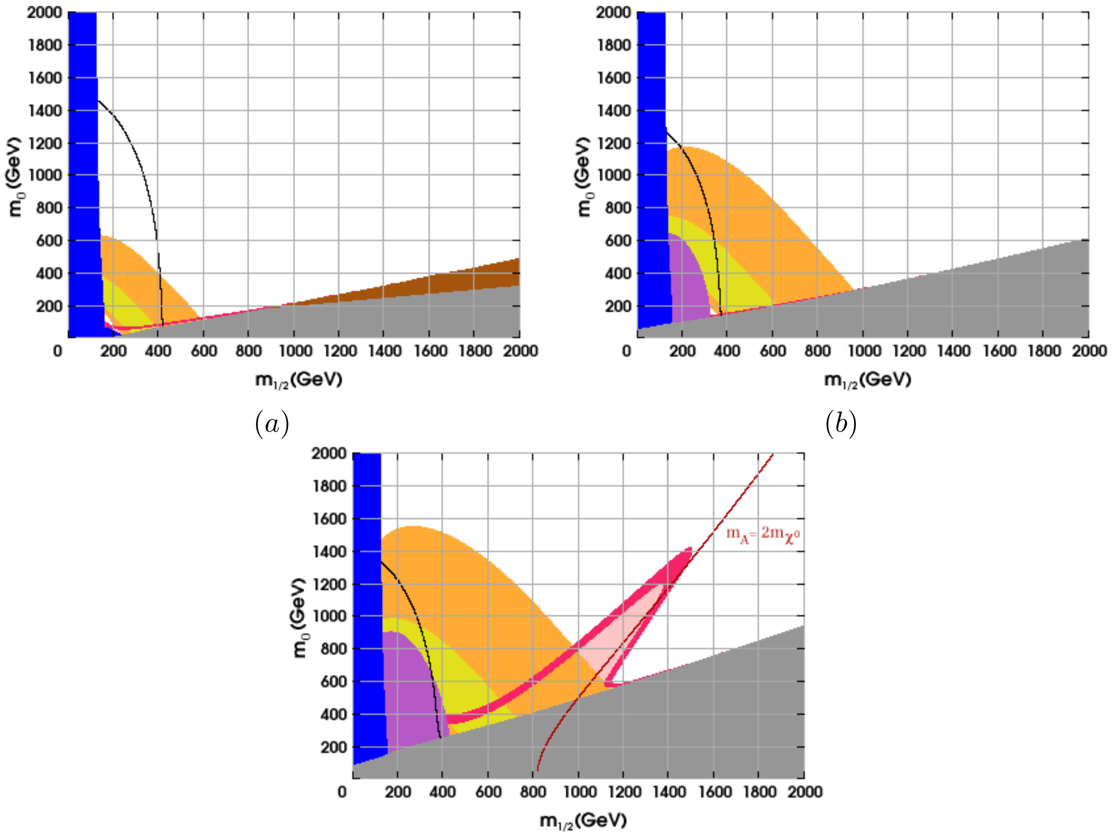
$$-\overline{m}^2_{H_2} \geq \frac{1}{2} m_Z^2 + \frac{1}{2} \mathrm{Re}\,\Pi^T_{ZZ} - \frac{\overline{m}^2_{H_1}}{\tan^2\beta}\,,$$

which must be satisfied in order to obtain $|\mu|^2 \geq 0$.

Since we have $\tan^2\beta \geq 100$, the $\overline{m}^2_{H_1}$ term is suppressed and therefore we require the $\overline{m}^2_{H_2}$ term to be very small or negative for successful EWSB. This can be achieved by large radiative corrections [60]. The one-loop RGE for $m^2_{H_2}$ in the third family approximation is [61]

$$16\pi^2 \frac{dm^2_{H_2}}{d\log Q} = 6|y_t|^2 \left(m^2_{H_2} + (m^2_{\tilde{Q}})_{33} + (m^2_{\tilde{u}_R})_{33} \right) + 6|a_t|^2 - 6g_2^2|M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2\,.$$

One can see from this that by decreasing the squark mass squared parameters $(m^2_{\tilde{Q}})_{33}$ and $(m^2_{\tilde{u}_R})_{33}$, $m^2_{H_2}$ will be driven to relatively higher values at low scales. As a result the range of m_0 for which the electroweak symmetry will be broken successfully is reduced for a given $m_{1/2}$. A correlated consequence is that the effects associated with the focus point region in the standard CMSSM are pushed to much lower values of m_0 and a new strip of acceptable



(c)

Figure 3: Same as figure 1, but with $m^2_{\tilde{Q}}(m_G) = m^2_{\tilde{u}_R}(m_G) = m^2_{\tilde{d}_R}(m_G) = m^2_{\tilde{L}}(m_G) = m^2_{\tilde{e}_R}(m_G) = m_0^2 \mathrm{diag}(1, 1, 1 - \delta m^2)$. In the new brown(dark grey) region in (a) the LSP is a smuon, not a stau.

relic density consistent with the constraints appears for $\tan\beta \gtrsim 30$ due to the increased Higgsino component of the LSP.

It should be noted that in this region the primary mechanism for neutralino annihilation is *not* annihilation to massive gauge bosons or coannihilation with charginos, but annihilation to $b\bar{b}$ and $\tau\bar{\tau}$ via an s-channel A^0. The region where annihilation to gauge bosons or chargino coannihilation dominates is closer to the boundary where $|\mu| = 0$ and here the relic density is too small to account for $\Omega_{CDM}h^2$. As one moves away from this boundary in the direction of increasing $m_{1/2}$ towards the region of favoured relic density two important things happen. Firstly, the Higgsino component of the neutralino decreases significantly. As a result, since the amplitude for annihilation to gauge bosons is proportional to the square of their coupling to the Higgsino component of the neutralino, the cross-section rapidly drops. This is not the case for the annihilation amplitude featuring an s-channel A^0 boson because it only depends linearly on the coupling to the Higgsino component. Secondly, the mass difference between the lightest chargino and the LSP neutralino increases and coannihilation quickly becomes negligible. Although not at the A^0 resonance, relative to the case of no sfermion splitting, the amplitude for annihilation via the A^0 is enhanced for two reasons:

JHEP08(2005)031

(i) Due to the larger Higgsino component of the neutralinos, their coupling to the A^0 is increased.

(ii) The mass difference $2m_{\tilde{\chi}_1^0} - m_{A^0}$ is far smaller than for the usual focus point region at large m_0, thus enhancing the propagator.

As $m_{1/2}$ is increased further, past the first band of acceptable relic density, there is a rise in $\Omega_{CDM}h^2$ caused by the decrease in Higgsino component of the LSP. However, in the case of $\tan\beta = 50$, $\Omega_{CDM}h^2$ quickly drops again as the A^0 resonance effects become important. This explains the existence of two bands of acceptable neutralino relic density in figure 2(c) below the A^0 resonance for $m_0 \gtrsim 1200$ GeV.

For $\tan\beta \gtrsim 50$ another effect comes into play. As well as decreasing with increasing $\tan\beta$, m_{A^0} decreases with decreasing third family sfermion soft masses. Therefore the zone of parameter space in which the A^0 resonance occurs is pushed to lower values of $m_{1/2}$. Moreover, this effect contributes to reason (ii) given in the previous paragraph. When $\tan\beta \gtrsim 50$, $\Omega_{CDM}h^2 \leq 0.0945$ for most of the region favoured by $(g-2)_\mu$ not excluded by other constraints. The reason for this reduction in m_{A^0} can be understood from the formula for $m^2_{A^0}$ from the EWSB conditions and the renormalisation group equations for $m^2_{H_1}$ and $m^2_{H_2}$.

From the EWSB requirements [61],

$$\begin{aligned} m^2_{A^0} &= \frac{1}{\cos 2\beta}\left(\overline{m}^2_{H_2} - \overline{m}^2_{H_1}\right) - m^2_Z - \mathrm{Re}\,\Pi^T_{ZZ}(m^2_Z) - \\ &\quad - \mathrm{Re}\,\Pi_{AA}(m^2_{A^0}) + \frac{t_1}{v_1}\sin^2\beta + \frac{t_2}{v_2}\cos^2\beta \end{aligned}$$

where Π_{AA} is the A^0 self energy. The dominant term is the one containing the soft Higgs masses so $m^2_{A^0} \sim m^2_{H_1} - m^2_{H_2}$. Comparing the relevant terms in the renormalisation group equations for these parameters [61]

$$\begin{aligned} 16\pi^2 \frac{dm^2_{H_2}}{d\log Q} &= 6|y_t|^2\left(m^2_{H_2} + (m^2_{\tilde{Q}})_{33} + (m^2_{\tilde{u}_R})_{33}\right) + \cdots \\ 16\pi^2 \frac{dm^2_{H_1}}{d\log Q} &= 6|y_b|^2\left(m^2_{H_1} + (m^2_{\tilde{Q}})_{33} + (m^2_{\tilde{d}_R})_{33}\right) + 2|y_\tau|^2\left(m^2_{H_1} + (m^2_{\tilde{L}})_{33} + (m^2_{\tilde{e}_R})_{33}\right) + \cdots \end{aligned}$$

one can see that, due to the large top Yukawa coupling, any change in the soft sfermion masses will have a larger effect on the renormalisation group equation for $m^2_{H_2}$ than for $m^2_{H_1}$. Indeed it is the terms proportional to $|y_t|^2$ that are mainly responsible for driving $m^2_{H_2} < m^2_{H_1}$ in the first place. Lowering the third family soft sfermion masses will reduce the difference $m^2_{H_1} - m^2_{H_2}$ and thereby lower m_{A^0}. Figure 4 shows the RG running of this difference from M_G to m_Z numerically for the three different sfermion soft mass matrices considered in this paper, for $\tan\beta = 50$ and typical values $m_0 = m_{1/2} = 500$ GeV. Also shown is the value of m_{A^0} in each case.

The opposite effects to those described above pertain to figure 3(c). There is no boundary where $|\mu|^2 = 0$ below values of $m_0 \sim 8\,\mathrm{TeV}$ or higher and the A^0 resonance

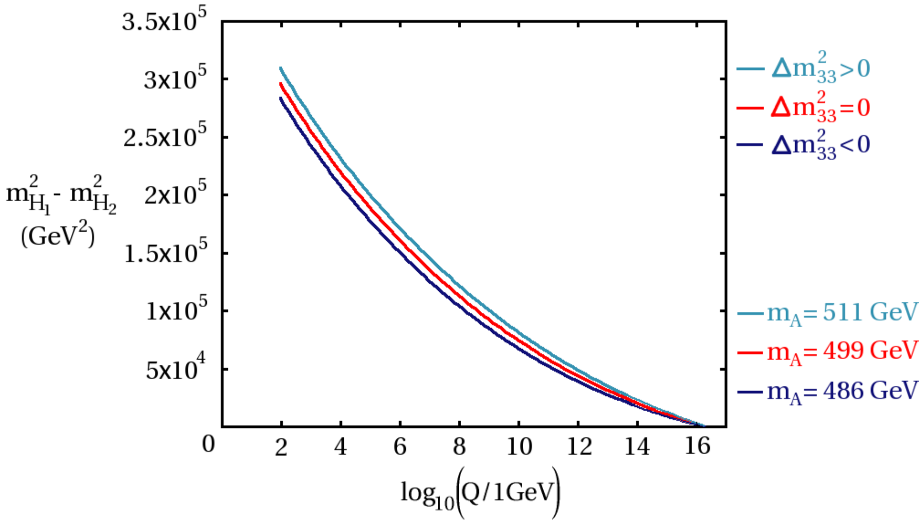


Figure 4: This plot shows renormalisation group evolution of the difference $m_{H_1}^2 - m_{H_2}^2$ from the GUT scale down to the EWSB scale for the three different boundary conditions on the sfermion soft mass matrices with $m_0 = m_{1/2} = 500$ GeV, $A_0 = 0$, $\tan\beta = 50$ and $\mu > 0$. The cyan(light) line corresponds to $\delta m_{\tilde{f}_{33}}^2(m_G) = m_0^2 \delta m^2$, the red(medium) line to $\delta m_{\tilde{f}_{33}}^2(m_G) = 0$, and the dark blue(dark) line to $\delta m_{\tilde{f}_{33}}^2(m_G) = -m_0^2 \delta m^2$

is slightly shifted towards higher $m_{1/2}$. As a result, the neutralino relic abundance at resonance is increased due to the larger values of μ and $m_{\tilde{\chi}_1^0}$ relative to the case with no sfermion splitting.

6. Summary and conclusions

Supersymmetric phenomenology is very sensitive to the soft SUSY breaking parameters which determine the spectrum of the new supersymmetric states. These parameters are strongly constrained by the need to avoid large flavour changing neutral currents and this has led to the construction of several distinct classes of model for supersymmetry breaking which are capable of solving the SUSY family problem. In this paper we have explored some of the implications of an alternative solution following from a non-abelian family symmetry. This is perhaps a more attractive solution than those based on gravity mediation, gauge mediation or anomaly mediation because the solution arises as a natural byproduct of a theory of fermion masses. In the case there is an SU(3) family symmetry the soft SUSY breaking masses of the three family members in a given representation of the Standard Model are degenerate up to SU(3) breaking effects and this is sufficient to avoid large FCNC. However this symmetry must be strongly broken to generate the third family of fermion masses and this inevitably leads to a breaking of the sfermion degeneracy, splitting the third family from the first two families. Although this breaking is small enough to avoid unacceptably large FCNC it does lead to significant changes in the phenomenology, particularly in the radiative generation of electroweak breaking and in the dark matter abundance following from the existence of a stable lightest supersymmetric particle. We have explored these effects in detail for the case that an underlying GUT guarantees the initial degeneracy of all the squarks and sleptons of a given family.

The main conclusion is that even the the small splitting of the degeneracy of the third family of the size indicated by the fermion mass structure leads to significant changes from the CMSSM phenomenology that has been widely used as a benchmark for future SUSY searches. In particular a reduction of the third family squark masses leads to a reduction in the radiative corrections that are needed to trigger electroweak breaking and this in turn extends the region excluded by the WMAP constraints. Associated with this is the fact that the region where the μ mass is small, close to the electroweak exclusion region, moves in the $(m_{1/2}, m_0)$ plane. In this region the Higgsino component of the LSP is enhanced and this significantly affects the LSP annihilation rate. This opens up a new region of parameter space where the LSP residual abundance is able to explain the dark matter abundance. It will be important to explore this region too in future searches for supersymmetry.

The effects explored here are the minimal ones to be expected in the case the family problem is solved by a non-abelian family symmetry. Although we have motivated the study in the context of a specific SU(3) family symmetry it is likely they have more general applicability as the magnitude of the effects follow from the need to generate the large masses of the third family of quarks and leptons and they are further constrained by the need to suppress FCNC. As we have discussed above, a non-abelian family solution to the family problem allows for even more significant changes from the CMSSM boundary conditions because, when the underlying GUT is broken, there may be significant splitting between different Standard Model representations provided each representation is separately nearly degenerate in family space. These effects will be explored elsewhere [62].

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